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1996 J. Phys. A: Math. Gen. 29 1897

(http://iopscience.iop.org/0305-4470/29/9/006)

J. Phys. A: Math. Gen. **29** (1996) 1897–1904. Printed in the UK

Effect of random impurities on fluctuation-driven first-order transitions

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Received 28 November 1995

Abstract. We analyse the effect of quenched uncorrelated randomness coupling to the local energy density of a model consisting of N coupled two-dimensional Ising models. For $N > 2$ the pure model exhibits a fluctuation-driven first-order transition, characterized by runaway renormalization-group behaviour. We show that the addition of weak randomness acts to stabilize these flows, in such a way that the trajectories ultimately flow back towards the *pure* decoupled Ising fixed point, with the usual critical exponents $\alpha = 0$, $\nu = 1$, apart from logarithmic corrections. We also show by examples that, in higher dimensions, such transitions may either become continuous or remain first-order in the presence of randomness.

The effect of quenched randomness coupling to the local energy density of a system which, in its absence, undergoes a continuous phase transition is well understood from the point of view of the renormalization-group version of the Harris criterion [1]. When the specific heat exponent α of the pure model is negative, weak randomness is irrelevant from the renormalization-group point of view, and the pure fixed point is stable. On the other hand, when $\alpha > 0$ it is relevant, and, at least when the crossover exponent α is small, it may be argued that the critical behaviour is controlled by a new, random, fixed point close by.

The effect on systems which undergo thermal first-order transitions is more dramatic. It was argued some time ago by Imry and Wortis [2] that, in two dimensions, such systems should always exhibit a continuous transition in the presence of such randomness. This is because the random impurities couple to the local energy density in much the same way that a random field couples to the local magnetization in an Ising system. In dimensions $d \leqslant 2$, the Imry–Ma argument [3] implies that such random fields should destroy the ordered phases at low temperature, and therefore also the first-order phase boundary between them. A similar argument, applied to randomness coupling to the local energy density, then implies that a non-zero latent heat is impossible in two dimensions in random systems whose pure versions exhibit such behaviour. This argument has been rediscovered and put on a rigorous basis by Aizenman and Wehr [4], and is supported by the phenomenological and approximate renormalization-group arguments of Hui and Berker [5]. Monte Carlo work of Chen *et al* [6] on the $q = 8$ state Potts model and of Domany and Wiseman [7] on the Ashkin–Teller and four-state Potts models supports this conclusion, and goes further: the continuous transition found by these workers exhibits critical exponents which are consistent with those of the pure Ising model, namely $\gamma/\nu \approx 1.75$, $\beta/\nu \approx 0.125$ and $\alpha \approx 0$. An argument explaining these findings has been put forward by Kardar *et al* [8]. They study the properties of an

0305-4470/96/091897+08$19.50

interface in the q-state random-bond Potts model at low temperatures. For $q \neq 2$ this has a branching structure, but the authors argue, on the basis of simplified recursion relations which are exact on a hierarchical lattice, that the critical behaviour where the interfacial free energy vanishes is governed by a zero-temperature fixed point (as in the random-field problem), and the Widom exponent μ which governs the vanishing of the surface tension is independent of q for sufficiently large q, being numerically consistent with the Ising value $\mu = 1$.

The first-order transition in the pure q-state Potts model is of mean-field type, that is, it is already predicted on the basis of mean-field theory. Such 'strong' first-order transitions are described within the renormalization group by zero-temperature, discontinuity fixed points, characterized by a relevant renormalization-group eigenvalue $y = d$ whose scaling field couples to the local energy density. Quenched randomness coupling to this has eigenvalue $d - 2(d - y) = d$, and is therefore also strongly relevant. It is thus difficult to treat the effects of such randomness systematically within a controlled renormalization-group calculation.

In this paper, by contrast, we study the effects of quenched randomness coupling to the local energy density on systems whose pure versions exhibit *fluctuation-driven* first-order transitions. These are transitions which are expected to be continuous on the basis of a mean-field analysis, but which are driven at first order by the fluctuation effects. In terms of the renormalization group, they are often characterized by so-called runaway behaviour, that is, the renormalization-group trajectories move out of the region in which the original perturbative calculation is valid. That, in itself, does not guarantee that the system in question undergoes a first-order transition, but often it is possible to argue that the trajectories then move into a region where mean-field theory is applicable, and which may then predict a first-order transition. The Imry–Wortis argument [2, 4] should, of course, apply equally well to systems exhibiting fluctuation-driven first-order transitions. However, the advantage of studying these from the renormalization group point of view is that it is possible to analyse them within a controlled perturbative scheme and to elucidate the nature of the fixed point which governs the continuous critical behaviour of the random system.

A simple example of a two-dimensional system which exhibits a fluctuation-driven first-order transition is that of N Ising models coupled through their local energy densities. Microscopically this may be represented in terms of a lattice model with N Ising spins $(s_1(r), \dots, s_N(r))$ at each site r of the lattice. The reduced Hamiltonian is

$$H = -K \sum_i \sum_{r,r'} s_i(r) s_i(r') - g \sum_{i \neq j} \sum_{r,r'} s_i(r) s_i(r') s_j(r) s_j(r') \tag{1}$$

where the sums over (r, r') are over nearest-neighbour sites. Such a model is self-dual on the square lattice, so that the critical coupling K_c may be found exactly. In the absence of randomness, the renormalization group equations on the critical surface have the form

$$\mathrm{d}g/\mathrm{d}\ell = (N-2)g^2 + \mathrm{O}(g^3) \,. \tag{2}$$

For $N = 2$, this vanishes, as expected since this case corresponds to the Ashkin–Teller model which exhibits a line of fixed points labelled by g [9]. When $N > 2$, however, initially small positive values of g flow out of the region of validity of the perturbative equation (2). When equation (1) is analysed within mean-field theory, the quartic term in the free energy remains positive if g is small, indicating a continuous transition, but, for sufficiently large g, it changes sign so that the mean-field transition becomes first-order. Since the renormalization group indicates that, no matter what the initial value of g, it should ultimately renormalize into this region, it implies that the transition should be first-order for *all* g, and is therefore of a fluctuation-driven nature for small g. In fact, on the critical surface, this model when expressed in terms of Ising fermions is nothing but the

Gross–Neveu model [10], which is believed to be massive for $N > 2$, corresponding to a finite correlation length. For $N \to \infty$ this model may also be solved exactly, with the same conclusion [11].

We now consider adding quenched randomness which couples to the local energy density. This may be done in a variety of ways, but, in order to focus on the universal properties of such a coupling, let us first rewrite (1) in a continuum notation in terms of the local energy density $E_i(r)$ of each Ising model. The Hamiltonian density, close to the critical point of the pure system, may then be written as

$$\mathcal{H} = \mathcal{H}_c + t \sum_i E_i - g \sum_{i \neq j} E_i E_j \tag{3}$$

where H_c is the fixed-point Hamiltonian and t is the temperature deviation from the critical point. Quenched randomness is now added by allowing $t \to t + \delta t(r)$, where $\overline{\delta t(r)} = 0$ and $\overline{\delta t(r) \delta t(r')} = \Delta \delta(r - r')$. Introducing n replicas $a = 1, \ldots, n$ and averaging over a Gaussian distribution, the replicated Hamiltonian density is

$$\mathcal{H} = \sum_a \mathcal{H}_c^a + t \sum_{i,a} E_i^a - g \sum_{i \neq j, a} E_i^a E_j^a - \Delta \sum_{i,j,a,b} E_i^a E_j^b \,. \tag{4}$$

Note that each term has a well-defined behaviour under the duality operation under which E_i^a reverses sign. The self-dual critical point therefore remains at $t = 0$ in this parametrization. It is possible to consider replica-coupling terms which break this duality symmetry, but they are all irrelevant close to the pure decoupled fixed point.

The perturbative renormalization group equations for the couplings follow, using standard methods. In general for a perturbed Hamiltonian density of the form $\mathcal{H} = \mathcal{H}_c + \sum_i g_i \Phi_i$, they have the form [12]

$$\mathrm{d}g_k / \mathrm{d}\ell = y_k g_k - \sum_{i,j} c_{ijk} g_i g_j + \mathrm{O}(g^3) \tag{5}$$

where y_i is the eigenvalue at the unperturbed fixed point, and c_{ijk} is the coefficient of Φ_k in the operator product expansion of Φ_i with Φ_j. In the present case, these are very easy to work out. Both the interaction terms in (4) have a similar form, and are in fact special cases of a very general model of Nn interacting Ising models, with a Hamiltonian density

$$\mathcal{H} = \mathcal{H}_c + \sum_{p \neq q = 1}^{Nn} G_{pq} E_p E_q \,. \tag{6}$$

The terms with $p = q$ are excluded since the operator product expansion of E_p with itself in the Ising model yields only the trivial identity operator. Normalizing the energy density so that $E_p \cdot E_{p'} = \delta_{pp'}$, the required terms in the operator product expansion are then

$$(E_p E_q) \cdot (E_{p'} E_{q'}) = \delta_{pp'} (E_q E_{q'}) + \text{permutations} + \cdots \tag{7}$$

from which follow the general renormalization group equations:

$$\mathrm{d}G_{pq} / \mathrm{d}\ell = -4(1 - \delta_{pq}) \sum_r G_{pr} G_{rq} + \mathrm{O}(G^3) \,. \tag{8}$$

Specializing these to the case at hand, we then find, in the limit $n \to 0$, the flow equations

$$\mathrm{d}g / \mathrm{d}\ell = 4(N-2) g^2 - 8 g \Delta + \cdots \tag{9}$$

$$\mathrm{d}\Delta / \mathrm{d}\ell = -8\Delta^2 + 8(N-1) g \Delta + \cdots \tag{10}$$

$$\mathrm{d}t / \mathrm{d}\ell = t\big(1 - 4\Delta + 4(N-1) g\big) + \cdots \,. \tag{11}$$

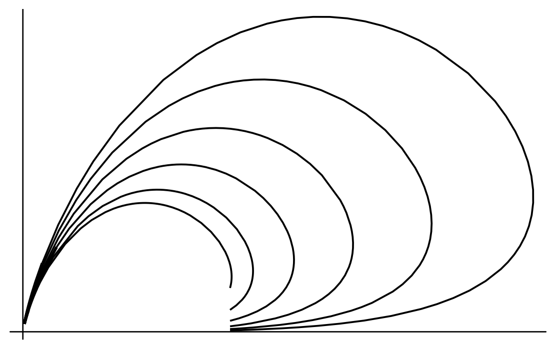

Figure 1. RG flows on the critical surface for the case $N = 3$. Several trajectories are shown, for different initial values of the randomness Δ at a fixed value of the coupling g between the Ising models. Although they initially flow towards strong coupling, they eventually curl back towards the pure Ising fixed point.

The last equation in fact follows from the second by the renormalization group version of the Harris criterion [13]. For the case $N = 2$ these equations are equivalent to those found for the random Ashkin–Teller model by Dotsenko and Dotsenko [14]. Remarkably, the flows in the critical surface obtained by solving these equations may be found in closed form for general N:

$$g = \text{constant} \times (\Delta/g)^{(N-2)/N}\, \mathrm{e}^{-2\Delta/Ng}\,. \tag{12}$$

These are shown in figure 1 for the case $N = 3$.

For $g = 0$ we find that the randomness is marginally irrelevant, consistent with the well known case of the random Ising model [15, 16]. In the absence of randomness, the flows for $g > 0$ run away to the first-order region, as discussed above. However, for any non-zero randomness the trajectories eventually curl around and approach the fixed point corresponding to N decoupled pure Ising models. Of course equations (9)–(11) are strictly valid only inside the perturbative region where the initial values of the parameters are small, but it is reasonable to expect that the topology of the renormalization group flows should persist at least in some finite region around the origin. This topology has two important consequences: first, as dictated by the Imry–Wortis argument, the transition has become continuous, and secondly, the asymptotic critical behaviour is that of the pure Ising model, similar to those cases discussed earlier. In fact, by integrating equation (11) for t it may be shown that the specific heat has a singularity of the form $A \ln\ln(1/t)$, just as for the random bond Ising model [15], but with an amplitude $A \propto \Delta^{-(N-2)/N}$. Flows of the type shown in figure 1 are very unusual as they violate the c-theorem [17]. Of course, that this can happen is a consequence of the $n \to 0$ replica limit.

It is instructive to extend the above analysis to dimensions $d = 2 + \epsilon$, since the Imry–Wortis argument leads to no definite conclusion in that case. The perturbative renormalization group equations become

$$\mathrm{d}g/\mathrm{d}\ell = \alpha_p g + \big(4(N-2) + 2b^2\big)g^2 - (8 - 4b^2)g\Delta + \cdots \tag{13}$$

$$\mathrm{d}\Delta/\mathrm{d}\ell = \alpha_p \Delta - (8 - 2b^2)\Delta^2 + 8(N-1)g\Delta + \cdots \tag{14}$$

$$\mathrm{d}t/\mathrm{d}\ell = t\big(d/(2-\alpha_p) - (4 - 2b^2)\Delta + 4(N-1)g\big) + \cdots\,. \tag{15}$$

The linear terms are a consequence of the fact that the specific heat exponent α_p of the pure model no longer vanishes: this determines the eigenvalues of g and Δ at this fixed point according to the Harris criterion. The parameter b is the operator product expansion coefficient appearing in $E_p \cdot E_q = \delta_{pq} + b\delta_{pq}E_p + \cdots$. With the energy density normalized in this way, it is universal, depending only on d, but it vanishes for $d = 2$ as a consequence of duality, and is therefore presumably small just above two dimensions. (In $d = 4$, at the Gaussian fixed point, $b = 2\sqrt{2}$.) When $g = 0$, the randomness is now relevant, and the critical behaviour is controlled by a non-trivial random fixed point at $\Delta = \mathrm{O}(\alpha_p)$ [13]. However, when $b \neq 0$, g is in fact relevant at this fixed point, and there exists another, more stable fixed point, which, for $b \ll 1$, is located at $g \approx b^2\alpha_p/16N$, $\Delta \approx \alpha_p/8$. When $g > 0$ initially, the trajectories move towards larger values of g before eventually curling around to finish at this new coupled random fixed point. Therefore this gives an example of a fluctuation-driven first-order transition in $d > 2$ dimensions, which is converted to a continuous transistion, as in $d = 2$. However, now the critical behaviour is controlled by a new, random, fixed point. Such a fixed point, if it persists as far as $\epsilon = 1$, would describe the random Ashkin–Teller model in three dimensions, at least for small values of g.

The above calculation breaks down near four dimensions (if not before), due to the proximity of the Gaussian fixed point. As a further example of what can happen to a fluctuation-driven first-order transition in $4-\epsilon$ dimensions, consider the well known problem of the $\mathrm{O}(N)$, or N-vector, model, with cubic symmetry breaking [18]. This model has N-component continuous spins $S_i(r)$, and the replicated Hamiltonian density is

$$\mathcal{H} = t\sum_{i,a}\left(S_i^a\right)^2 + u\sum_{i,j,a}\left(S_i^a\right)^2\left(S_j^a\right)^2 + v\sum_{i,a}\left(S_i^a\right)^4 - \Delta\sum_{i,j,a\neq b}\left(S_i^a\right)^2\left(S_j^b\right)^2. \quad (16)$$

For $u \ll v$ this may be viewed as a continuous spin version of (4), with $g = -u$. However, in the absence of randomness this model also possesses an $\mathrm{O}(n)$ fixed point (which is absent for $d = 2$) and a cubic fixed point where both u and v are non-zero. The perturbative renormalization group equations may be found from the operator product expansion as above:

$$\mathrm{d}u/\mathrm{d}\ell = \epsilon u - 8(N+8)u^2 + 8N\Delta^2 - 48uv + \cdots \quad (17)$$

$$\mathrm{d}v/\mathrm{d}\ell = \epsilon v - 72v^2 - 96uv + \cdots \quad (18)$$

$$\mathrm{d}\Delta/\mathrm{d}\ell = \epsilon\Delta - 16(N-2)\Delta^2 - 16(N+2)u\Delta - 48v\Delta + \cdots. \quad (19)$$

When $\Delta = 0$ these exhibit runaway behaviour to the first-order region where u and v are large and negative, if the initial value of v is sufficiently negative (for $N > 4$, when the cubic fixed point is in the quadrant with $v > 0$, this requires only that $u > 0$ and $v < 0$ initially). However, since Δ does not enter the flow equation for v, this catastrophe still occurs in the presence of randomness. We conclude that quenched randomness does not change the order of the transition in this case. This is, of course, quite consistent with the Imry–Wortis argument, which does not rule out either behaviour for $d > 2$.

The case where $u < 0$ and $v > 0$ corresponds to the same example of N coupled Ising models as before, this time near four dimensions. Solving the renormalization group equations for v near the decoupled Ising fixed point, we find (using g rather than u)

$$\mathrm{d}g/\mathrm{d}\ell = \tfrac{1}{3}\epsilon g + 8Ng^2 + 32g\Delta + \cdots \quad (20)$$

$$\mathrm{d}\Delta/\mathrm{d}\ell = \tfrac{1}{3}\epsilon\Delta + 16(N-2)g\Delta + \cdots. \quad (21)$$

When $g = 0$, these equations have no perturbative fixed point, despite the fact that Δ is relevant. This is the well known problem of the random Ising model near $d = 4$, and it is cured in a higher-order calculation [19], when a term $\mathrm{O}(\Delta^3)$ appears on the right-hand side of (21), giving an $O(\epsilon^{1/2})$ fixed point. However, it may be seen from the structure of the other terms in (20) and (21) that this cannot cure the runaway behaviour which occurs once the coupling g is initially non-zero. We conclude that the fluctuation-driven first-order transition probably persists in this case.

As a final example, we may quote the case of the complex $O(N)$ model near four dimensions, coupled to a long-range $U(1)$ gauge field, known as the Abelian Higgs model for the case $N = 1$. This was argued long ago to undergo a fluctuation-driven first-order transition near four dimensions for sufficiently small N [20]. The effect of quenched random impurities was studied by Boyanovsky and Cardy [21], who found that for sufficiently weak randomness the first-order nature of the transition persists, while for stronger randomness the trajectories spiral in towards a new random fixed point, corresponding to a continuous transition.

To summarize, we have given examples of how quenched randomness coupling to the local energy density converts a fluctuation-driven first-order transition into a continuous one for $d = 2$, consistent with the Imry–Wortis argument, and of how this may or may not happen when $d > 2$.

We conclude with a discussion of the conjecture that all random critical behaviour in two dimensions is Ising-like. This is based on numerical results on the random Ising model [22–24], the Ashkin–Teller model and the four-state Potts model [7]† (all of which exhibit continuous transitions in the absence of randomness), and the 8-state Potts model (which is first-order in its pure version.) It is backed up by the interface arguments of Kardar *et al* [8], which suggest that the Widom exponent for the random q-state Potts model is independent of q for sufficiently large q. (A similar lack of dependence on q has been argued for in the case of random Potts spin chains [26]; however, their critical behaviour is rather different in nature from that of the present case.)

The conjecture in the case of the Ising model and the Ashkin–Teller model (close to the decoupling point) agrees with the results of a perturbative renormalization group analysis [14, 16] and with our analysis above: the renormalization group trajectories curl around and end up at the Ising fixed point, giving Ising exponents, but with logarithmic (or log–log) modifications. However, a similar analysis [27, 28] applied to the random q-state Potts model for $q > 2$ indicates the existence of a new random fixed point whose critical exponents are not Ising-like, but depend on q. This analysis is valid only when $q - 2$ is small, but is consistent with earlier renormalization group results [29] for $q = 3$.

We have not been able to resolve this discrepancy, but would venture a few remarks which, in fact, may seem to confuse the situation further:

(i) The perturbative renormalization group arguments work with a distribution of randomness which is *self-dual*, corresponding on the lattice, for example, to an equal distribution of strong and weak bonds of strengths K and K^* which are dual to each other. Within the perturbative scheme, this is justified, since it may be argued that weak randomness which violates self-duality is irrelevant in the renormalization group sense.

(ii) However, the interfacial analysis of Kardar *et al* [8], which treats horizontal and vertical bonds on a quite different footing, cannot, by its nature, respect the duality properties of the model. Indeed, these authors find it necessary to include negative bonds in the model to access their zero-temperature fixed point, which are excluded in any self-dual formulation of the problem. The only zero-temperature fixed point in the self-dual

† There is also claimed experimental evidence for the four-state Potts model [25]; however, the randomness discussed there would appear to favour one sublattice rather than another, and therefore should couple to the order parameter, corresponding to the random-*field* problem.

random model is the percolation point.

(iii) This leads to the picture that the critical behaviour controlled by a zero-temperature fixed point, discussed by Kardar *et al* [8], and that found in the perturbative renormalization group of Ludwig [27], are simply different and correspond to strong non-self-dual randomness and to weak self-dual randomness, respectively. However, the numerical results for the $q = 4$ and $q = 8$ Potts models, which appear to find Ising-like exponents independent of q, use *self-dual* randomness in order to locate the critical point precisely. They also consider different strengths of randomness, with no appreciable difference in their results.

(iv) One would expect critical behaviour controlled by a zero-temperature fixed point to exhibit hyperscaling violation, as in the random field problem. However, the exponents found in the numerical work for $q = 8$ are consistent with hyperscaling, that is, with a conventional, finite-temperature fixed point as found in the perturbative renormalization group approach.

Whatever the resolution of this problem, it cannot be that all the universal properties of the random q-state Potts model are independent of q, even if the exponents are. This is because this critical point separates a q-fold degenerate ordered phase from a non-degenerate disordered phase, and this degeneracy must reflect itself in the fluctuation contribution to the free energy near the critical point, even if the exponents are Ising-like. This may be seen in the example of N coupled Ising models discussed in this paper: although the relevant critical fixed point is Ising-like, it in fact corresponds to N decoupled Ising models, not just one. This will reflect itself in universal amplitude ratios which involve the free energy. However, because of the expected logarithmic corrections, these may be difficult to analyse from numerical data. A cleaner test should be through the value of the effective central charge (which measures the finite-size scaling behaviour of the quenched free energy [30, 31]. In our example, this is $c = \frac{1}{2}N$, and does depend on N. It would be very interesting to compute this for the random q-state Potts model. To how many decoupled Ising models does the random q-state Potts model correspond at criticality, if indeed its behaviour is Ising-like?

After this work was completed, the author's attention was drawn to a recent paper [32] in which it is shown that random impurities act to restore the continuous nature magnetic system coupled to elastic degrees of freedom.

Acknowledgments

The author acknowledges useful discussions with E Domany, M Lassig, S Majumdar and S Wiseman, and thanks the Weizman Institute, where this work was initiated, for its hospitality. This work was supported in part by EPSRC grant GR/J78327.

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